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Variational Solution of Microwave Circuits and Structures

S. Tsitsos, N. Karamitsos, B. M. Dillon, and A. A. P. Gibson

Abstract— **A unified variational formulation for microwave planar transmission lines and lumped-element impedances is developed and applied to an isolated stripline power splitter. Scattering parameters are calculated via the transfinite-element method and the numerical results are corroborated by three-port experimental measurements. The microwave impedance of a thin-film isolation resistor is separately measured and included in the model.**

I. Introduction

Stripline power splitters with the appropriate power split ratio are cascaded together to produce the desired operative aperture field distribution in the corporate feeds of radar antenna systems [1]. Any distortion of the aperture excitation is minimized by using isolation load resistors suspended between coupled striplines to absorb reflected power [1]–[3]. This is one example of how highly irregular electromagnetic structures are often combined with lumped element components. Other examples range from varactor-tuned oscillators to high-directivity, capacitor-loaded directional couplers.

A new method that uses the variational solution of electrical circuits and fields is presented to cater for this class of problem. The transfinite-element method of Csendes and Lee [4] is extended to include lumped-element impedances and applied to an isolated stripline power splitter. Measured impedance data for a thin-film resistor is included in the model. Three-port scattering parameters and field plots are presented for an equal split junction. Experimental measurements are superimposed and compare well with the calculations. The amplitude of the TE_{10} higher-order mode at the port reference planes has also been calculated.

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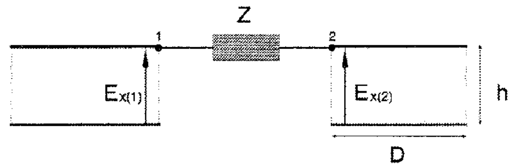

Fig. 1. Schematic of two planar waveguides with top and bottom electric walls and magnetic sidewalls connected via a lumped element impedance Z.

II. Variational Formulation

Variational principles for electromagnetic resonators and waveguides were first introduced by Berk [5]. These expressions are often used with finite-element methods to predict modal hierarchy and scattering parameters in waveguides and junctions [4], [6]. Stationary energy methods can also be applied to electrical lumped-element networks [7]. Recently, a formal procedure describing the variational solution of linear and nonlinear circuits has been enunciated [8]. These methods are extended here to include interconnection with distributed planar transmission lines. The geometry in Fig. 1 depicts two planar waveguides connected via a lumped-element impedance Z. The power dissipated in this impedance is a function of the nodal terminal voltages $hE_{x(1)}$ and $hE_{x(2)}$ and is written as

$$P(E_x) = \frac{|E_{x(1)} - E_{x(2)}|^2 h^2}{Z}. \tag{1}$$

Following Csendes and Lee the variational functional for the planar waveguide sections is given by

$$F(E_x) = \frac{h}{j\omega\mu}\left[\int\!\!\int_A (|\nabla E_x|^2 - \omega^2\epsilon\mu|E_x|^2)\,dA - \sum_{i=1}^{P}\oint_{\Gamma_i} E_x^*\frac{\partial E_x}{\partial n}\,d\Gamma\right]. \tag{2}$$

The area integral refers to the layout of a planar geometry and represents the stored electromagnetic energy. The line integral represents the power flow onto each port p. The permittivity, permeability, and angular signal frequency are denoted ϵ, μ, and ω, respectively. For planar waveguides interconnected or terminated by Q-lumped element impedances, (1) and (2) are combined as [9]

$$F(E_x) = \sum_{q=1}^{Q}\frac{|E_{x(i)} - E_{x(k)}|^2 h^2}{Z_q} + \frac{h}{j\omega\mu}\left[\int\!\!\int_A (|\nabla E_x|^2 - \omega^2\epsilon\mu|E_x|^2)\,dA - \sum_{i=1}^{P}\oint_{\Gamma_i} E_x^*\frac{\partial E_x}{\partial n}\,d\Gamma\right] \tag{3}$$

where each impedance Z_q is connected between nodes i and k of a finite-element mesh.

III. Stripline Formulation

The transfinite-element method described by [4] uses a planar waveguide transformation to model microstrip-like transmission lines. An efficient two-dimensional analysis of three-dimensional microstrip discontinuities and junctions then follows. Triplate stripline can be transformed into two parallel plate waveguides one on top of the other. Each waveguide has top and bottom electric walls and magnetic

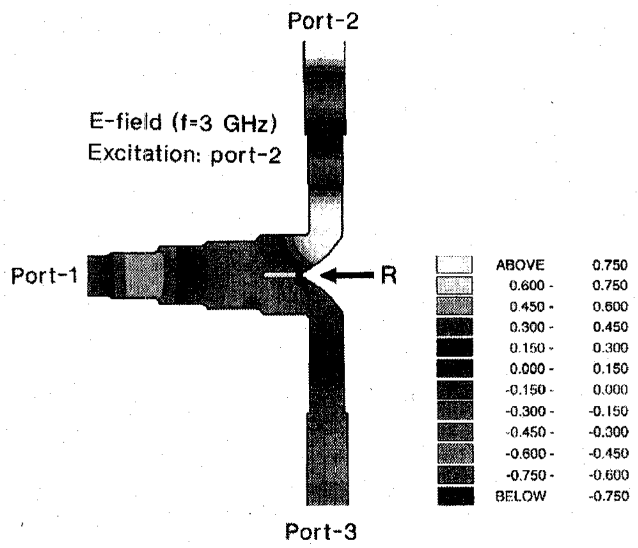

Fig. 2. Layout of an equal split isolated stripline power divider with the electric field E_x superimposed at 3 GHz for a wave excited at port 2 ($b = 12.7$ mm, $t = 0.127$ mm and $\lambda/4$ sections are 25 mm long).

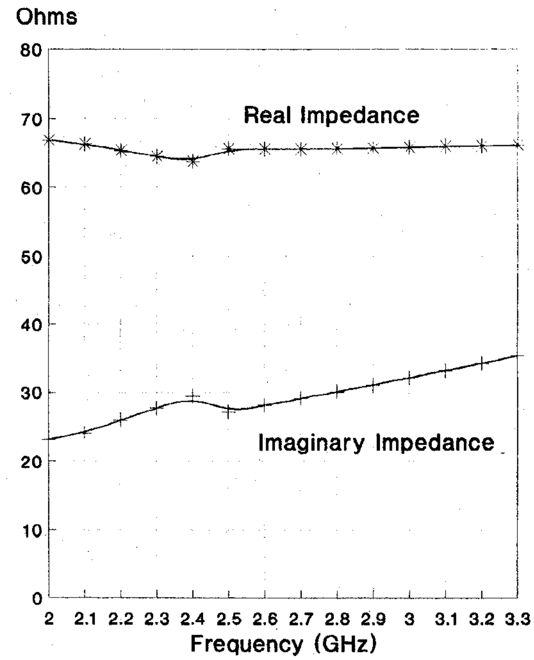

Fig. 3. Microwave impedance, measured using a thru reflect line technique, for a thin-film isolation resistor.

side walls. The modified width D, shown in Fig. 1, is related to the stripline central conductor of width w and thickness t by [10]

$$D = w + \frac{2b}{\pi} ln2 + \frac{t}{\pi}\left[1 - \ln\left(\frac{2t}{b}\right)\right] \tag{4}$$

where b is the spacing between the ground planes of the stripline. Only one of the parallel plate waveguides has to be modeled as the power flow through each waveguide is identical. In the functional (3) the height (h) must be replaced by $b/2$ and the integrals multiplied by 2 to cater for the parallel waveguides. Functional (3) takes its final form for air filled stripline as

$$F(E_x) = \sum_{q=1}^{Q} \frac{|E_{x(i)} - E_{x(k)}|^2}{\frac{Z_q}{b/4}} + \frac{1}{j\omega\mu_o}\left[\int\!\!\int_A (|\nabla E_x|^2 - \omega^2 \epsilon_o \mu_o |E_x|^2)\, dA - \sum_{i=1}^{P} \oint_{\Gamma_i} E_x^* \frac{\partial E_x}{\partial n}\, d\Gamma\right]. \tag{5}$$

IV. Numerical and Experimental Results

Isolated stripline power dividers are widely used in corporate feed antenna networks [1]. An equal-split arrangement is presented in Fig. 2. A microwave resistor R is mounted at the splitting junction in order to provide good isolation. As a preamble to modeling the isolated power divider, it was necessary to experimentally determine the lumped element impedance of the thin film isolation resistor. Fig. 3 illustrates how the resistor exhibits a substantial reactive component as well as a variation over the frequency range of interest.

The impedance data of Fig. 3 and the functional (5) were applied to the planar waveguide layout illustrated in Fig. 2. Lumped-element coupling capacitors between planar waveguides can be used to model coupled line effects [11]. The transfinite-element electric field distribution at 3 GHz is superimposed for an excitation wave at port-2. The S-parameter calculations over the operating frequency range for this type of divider are compared with experimental results in Figs. 4 and 5. Finally, the amplitude of the TE_{10} higher-order mode at the port reference planes has been calculated for an excitation wave

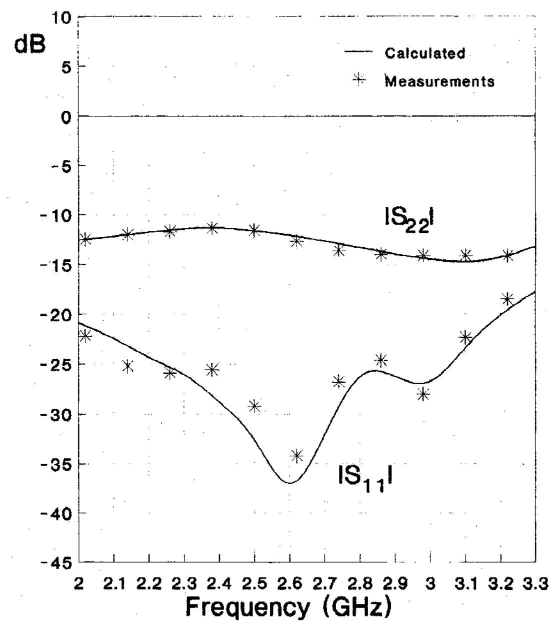

Fig. 4. Transfinite element and experimental measurements for $|S_{11}|$ and $|S_{22}|$ of an equal-split power divider.

incident at port-2. In Fig. 6 a significant amount of power is coupled to this mode at port-1, but very little at ports 2 and 3 due to higher impedance lines.

V. Conclusion

A unified variational method has been presented to cater for combined lumped element and electromagnetic field problems. Experimental measurements corroborate transfinite element calculations for an isolated stripline power splitter. This approach can be applied to a wide range of microwave hybrids.

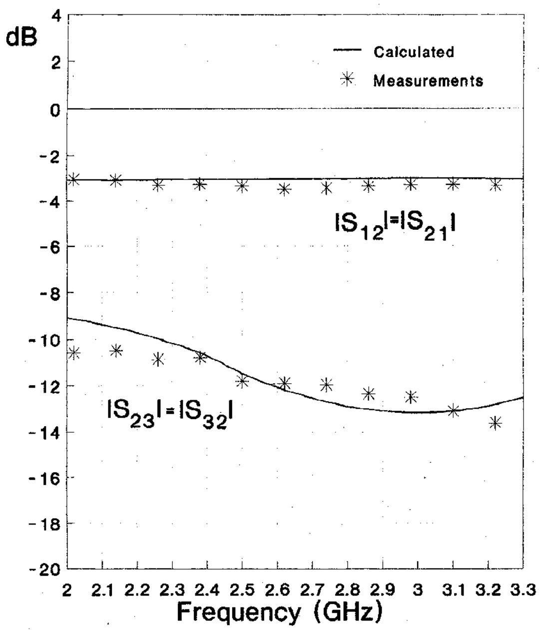

Fig. 5. Transfinite-element and experimental measurements for $|S_{12}|$, $|S_{21}|$, $|S_{23}|$, and $|S_{32}|$ of an equal-split power divider.

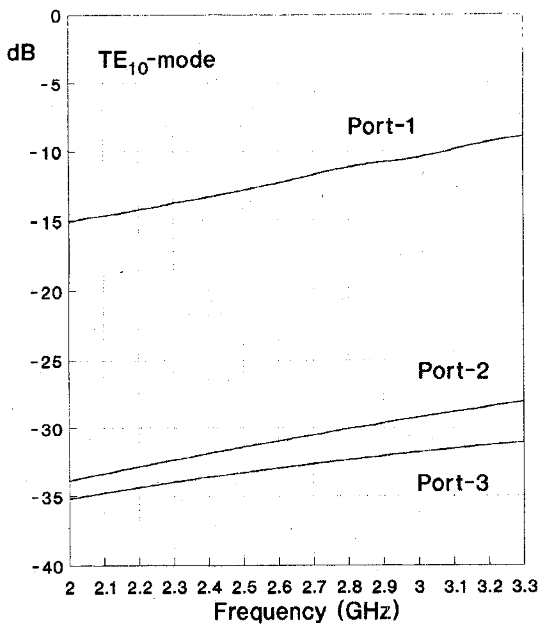

Fig. 6. Calculated amplitude of the TE_{10} mode at each port of an equal-split power divider for an excitation wave at port-2.

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An Efficient Synthesis Technique of Tapered Transmission Line with Loss and Dispersion

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***Abstract*—A sythesis technique of lossy and dispersive tapered transmission line is newly presented that extends lossless cases suggested by Klopfenstein [1] and others [2]–[4]. A special optimization process based on the Fourier transform pair [5] and generalized Taylor's procedure [6] is performed to extract the exact null points of lobe-like frequency response in terms of the input reflection coefficient of lossy-tapered line in which the loss may be frequency dependent and distance dependent. The theory is verified by evaluation of a synthesized microstrip taper profile in the lossy case and is expected to be helpful for design of tapered line in the high-frequency microwave integrated circuits (MIC's) with loss.**

I. INTRODUCTION

The tapered transnsmission line has been widely used in monolithic microwave/millimeter-wave integrated circuits (MMIC's) and high clock rate digital integrated circuits for the impedance transformation.

So far, no accurate synthesis for a specified frequency response representing input reflection coefficient has been given for lossy tapered transmission lines. Consider a tapered line, supporting a non-TEM mode, which is used as a transformer to match a line of impedance to a load of impedance (Fig. 1). It is well known that the input reflection coefficient for a tapered line is expressible to good approximated solution (for the case of $\rho \ll 1$) of the Riccati equation [1], [5]

$$\rho_i(\omega) = \int_0^L \frac{d \ln \overline{Z}(\omega, z)}{dz} \cdot \exp\left[-2\int_0^z \alpha(\omega, z')\, dz'\right]$$

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